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IDENTIFICATION OF HEAT TRANSFER BETWEEN THE CASTING AND THE MOLD IN INGOTLESS ROLLING

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A method is proposed for determining the heat flux inside a roll-mold on the basis of solution of the inverse heat-conduction problem by the gradient method.

The study of casting processes in a roll-type mold in order to select efficient production schemes entails the development of a set of mathematical models capable of predicting the thermal and thermal-stress states of equipment and products [1]. As is known [2, 3], the most important factor which affects solidification is heat transfer between the casting and the mold. At the same time, direct measurement of temperatures and heat fluxes on the mold surface is not possible because the transducers fail from thermal and mechanical loads [4]. It was proposed in [5] that the contact temperature be determined by means of a so-called natural thermocouple. Here, the contacting bodies themselves act as the thermoelectrodes. However, such a method is not sufficiently reliable and, moreover, does not permit consideration of the temperature distribution along the contact. Given these circumstances, it is best to obtain temperature measurements at internal points and to use inverse-problem methods to establish the thermal parameters on the contact surface [6].

A sketch of the equipment used for casting in a roll-type mold is shown in Fig. 1. Since the radius of the roll is considerably smaller than its length, we will assume that there is no heat transfer in the axial direction. Then the thermal problem for quasisteady operation of the roll is described as follows in cylindrical coordinates (ρ, φ) :

$$\bar{v}\,\frac{\partial T}{\partial \varphi}=\frac{1}{\rho}\,\frac{\partial}{\partial \rho}\left(\rho\,\frac{\partial T}{\partial \rho}\right)+\frac{1}{\rho^2}\,\frac{\partial^2 T}{\partial \varphi^2}, \tag{1}$$

$$0<\varphi<2\pi,\ \rho_0<\rho<1,$$

$$\left.\frac{\partial T}{\partial \rho}\right|_{\rho=\rho_0}=\frac{\alpha R}{\lambda}(T|_{\rho=\rho_0}-T_{\text{coo}}), \tag{2}$$

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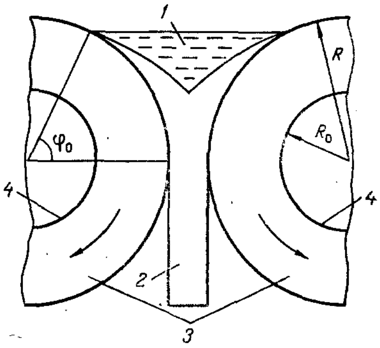

Fig. 1. Unit for casting in a roll-type mold: 1) melt; 2) casting; 3) roll; 4) surface undergoing cooling.

$$\left.\frac{\partial T}{\partial \rho}\right|_{\rho=1} = \frac{R}{\lambda} q(\varphi), \tag{3}$$

$$T|_{\varphi=0} = T|_{\varphi=2\pi}, \tag{4}$$

$$\left.\frac{\partial T}{\partial \varphi}\right|_{\varphi=0} = \left.\frac{\partial T}{\partial \varphi}\right|_{\varphi=2\pi}. \tag{5}$$

The coordinate φ in mathematical model (1-5) is reckoned from the beginning of contact of the role with the forming skin. Ignoring heat losses from the roll to the air outside the contact arc, we obtain q = 0 at $\varphi_0 < \varphi < 2\pi$.

The solution of thermal problem (1-5) should reduce to establishing a cause-and-effect relationship between the limiting heat flux and the temperature field inside the roll:

$$T(\varphi, \rho) = T_{\text{coo}} + \int_0^{\varphi_0} q(\varphi') G(\varphi - \varphi', \rho)\, d\varphi'. \tag{6}$$

Equation (6) in turn makes it possible to use the known temperature $T(\varphi, \rho)$ to establish the function $q(\varphi)$ and to thus identify the contact heat transfer between the casting and the roll.

A sizable number of studies, such as [7-11], has been devoted to the problem of determining the temperature field in a rotating roll. According to [10, 11], at high speeds of rotation, the temperature field in a cylindrical body is represented by the sum

$$T(\varphi, \rho) = \bar{T}(\rho) + \tilde{T}(\varphi, \rho), \tag{7}$$

in which the constant (i.e. independent of the angle φ) component $\bar{T}$ is determined by the total heat flux to the roll, while the periodic component $\tilde{T}$ penetrates the roll to an insignificant depth. Thus, we can consider the temperature field to be two-dimensional. Proceeding on this basis, we can easily obtain expressions for the constant and periodic components of the Green function:

$$\bar{G}(\xi) = \frac{R}{2\pi\lambda}\left(\ln\frac{1-\xi}{\rho_0} + \frac{\lambda}{\alpha R_0}\right), \tag{8}$$

$$\tilde{G}(\varphi, \xi; \bar{v}) = \frac{R}{\lambda\pi}\left[F(\varphi, \xi) + F(\varphi - 2\pi, \xi) - \frac{1}{2\pi}\int_{-2\pi}^{2\pi} F(\varphi', \xi)\, d\varphi' + \right.$$
$$\left. + \frac{1}{\sqrt{2\bar{v}}}\sum_{k=1}^{\infty}\left(\frac{1}{\sqrt{k+\varphi/2\pi}} - 2\sqrt{k+1} + 2\sqrt{k}\right)\right], \tag{9}$$

where $F(\varphi, \xi) = \exp\left(\frac{\bar{v}}{2}\varphi\right) K_0\left(\frac{\bar{v}}{2}\sqrt{\varphi^2+\xi^2}\right)$; $K_0(z)$ is a modified Bessel function; $\xi = 1 - \rho$ is the relative distance from the surface of the roll. Equation (8) is exact, while (9) is approximate with an error no greater than 1% at $\bar{v} > 100$, $\xi < 0.05$.

Then the connection between the limiting heat flux and the temperature field in the roll can be written in the form:

$$T(\varphi,\ \xi) = T_{\text{coo}} + \frac{Q}{2\pi\lambda}\left(\ln\frac{1-\xi}{\rho_0} + \frac{\lambda}{\alpha R_0}\right) + \int_0^{\varphi_0} q(\varphi')\tilde{K}(\varphi-\varphi',\ \xi)\,d\varphi', \tag{10}$$

where

$$\tilde{K}(\varphi,\ \xi) = \begin{cases}\tilde{G}(\varphi,\ \xi;\ \bar{v}),\ \varphi \geqslant 0,\\ \tilde{G}(\varphi+2\pi,\ \xi;\ \bar{v}),\ \varphi < 0;\end{cases}$$

$Q = R\int_0^{\varphi_0} q(\varphi')\,d\varphi'$ is the heat flux per unit length of the roll.

Equation (10) can be simplified by considering the fact that the roll contacts with the casting on a section which is small compared to the circumference of a circle ($\varphi/2\pi \lesssim 0.1$). As a result, the expression for the temperature field in the surface region of the roll is written in the form

$$\frac{\lambda\pi}{R}[T(\varphi,\ \xi) - T_0(\xi)] = \int_0^{\varphi} q(\varphi')\exp\left[\frac{\bar{v}}{2}(\varphi-\varphi')\right]K_0\left[\frac{\bar{v}}{2}\sqrt{(\varphi-\varphi')^2+\xi^2}\right]d\varphi' - \varepsilon_q(\varphi), \tag{11}$$

where $T_0(\xi) = T(0,\ \xi)$ is the temperature at the inlet of the roll-casting contact zone at the depth ξ; $\varepsilon_q(\varphi)$ is a correction for the periodicity of the process which causes the right side of (11) to vanish at $\varphi=2\pi$. However, the value of $\varepsilon_q(\varphi)$ is no greater than 1% of the first term for small values of φ and can be discarded.

Since the kernel of Eq. (11) decreases with an increase in ξ and $\bar{v}$, it is logical to propose that thermal boundary perturbations will decay with increasing depth from the roll surface. Thus, in the solution of the inverse boundary-value problem in the formulation (11), small oscillations in inlet temperature may destabilize the heat flux being established. The error of the inverse problem can be evaluated quantitatively by the method proposed in [6]. For example, if a sinusoidal change in temperature over time with the frequency ω_0 is assigned on the surface of a semi-infinite body, then the thermal oscillations will decrease with increasing depth by the law $\exp(-x\sqrt{\omega_0/2a})$. The character of change in the amplitude of the temperature oscillations is similar for formulation (11), i.e. is proportional to $\exp(-\xi\sqrt{\bar{v}\Omega/2})$. Here, Ω is the frequency of the oscillations with respect to the angular coordinate φ. This fact means that the heat sensor must be positioned as close to the roll surface as possible.

Thus, in solving the inverse problem, we need to use a regularizing algorithm to determine the function $q(\varphi)$ from Eq. (11). This can be done by introducing an extremal formulation of the problem and taking advantage of the iterative principle of regularization [6]:

$$\overset{(k+1)}{q}(\varphi) = \overset{(k)}{q}(\varphi) + \Delta\overset{(k)}{q}(\varphi),\ k = 0,\ 1,\ 2,\ \ldots, \tag{12}$$

where the correction $\Delta\overset{(k)}{q}(\varphi)$ is calculated from the condition of the greatest reduction of the below objective functional in each iteration

$$J(q) = \int_0^{\varphi_0}[T(\varphi,\ \xi) - T^*(\varphi)]^2 d\varphi. \tag{13}$$

In Eq. (13), the value of ξ is fixed and is determined by the depth of the heat sensor.

Assigning the increments $\delta q(\varphi)$ of the sought function $q(\varphi)$ we can obtain a formula for calculating the gradient of the functional

$$J_q'(\varphi) = 2\int_{\varphi}^{\varphi_0}[T(\varphi',\ \xi) - T^*(\varphi)]\exp\left[\frac{\bar{v}}{2}(\varphi'-\varphi)\right]K_0\left[\frac{\bar{v}}{2}\sqrt{(\varphi'-\varphi)^2+\xi^2}\right]d\varphi'.$$

The actual initial data often contains high-frequency components from both the noise of the measuring equipment and from thermal fluctuations. In the present case, the presence of these components is connected with the freezing of the skin and is governed to a

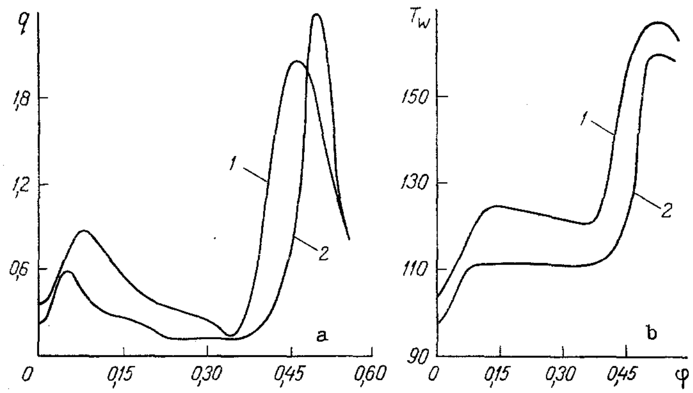

Fig. 2. Distribution of the boundary value of heat flux (a) and the corresponding roll-surface temperature (b) over the arc of contact with the casting (zeroth initial approximation, iterations stopped with an additional measurement): 1) $\omega = 0.485$; 2) $0.833\ \sec^{-1}$. q, W/m^2; T_W, °C; φ, rad.

large extent by the hydrodynamics of the melt and the microstructure of the forming skin. It is important that these fluctuations do not affect the position of the solid-liquid phase boundary. However, in the solution of the inverse problem, they may lead to oscillations of the sought function. The use of gradient methods of functional minimization makes it possible to circumvent this difficulty by stopping the iterations before signs of instability of the function begin to appear. A rigorously substantiated criterion for terminating regularizing gradient algorithms is the error criterion proposed in [12]. However, its successful use requires having a fairly accurate estimate of the random error of the inlet temperatures. This requirement is often not met in practice. An alternative method of stopping the iteration is to stop it after an additional measurement is made [13]. The use of this method in the solution of the inverse problem for a rotating cylindrical body should prove especially effective, since its realization in the present case does not necessarily require the installation of a second transducer. Readings from the thermocouple for different roll rotations can be used as the main and auxiliary input data. All periodic processes should obviously afford such an advantage.

Figure 2 shows results of analysis of experimental data on the casting of lead on an ingot-less rolling unit in the laboratory of contact heat transfer of the Mogilev branch of the Physicotechnical Institute, Academy of Sciences of the Belorussian SSR. Roll-molds made of steel 45 ($R = 0.1$ mm, $R_0 = 0.05$ m) were cooled with water at a temperature of 40°C ($\alpha = 1080\ W/(m^2 \cdot K)$).

The temperature at a point of the roll 1 mm from the surface was measured with a Chromel—Alumel thermocouple (channel diameter 1.2 mm). Thermoelectrodes 0.2 mm in diameter were welded to the bottom of the channel and insulated with fluoroplastic. The electrical signal from the sensor was transmitted through a mercury pickup and recorded on an oscillograph.

The disturbance of the thermophysical uniformity of the roll due to the presence of the thermocouple caused a distortion of the temperature field which reached 10-20% [14]. Failure to allow for this leads to a systematic error in the established heat flux on the roll surface and overestimation of the degree of reduction of the casting — which determines the quality of the resulting strip. It should also be noted that the distortion of the temperature field is determined not only by the geometric and thermophysical parameters of the sensor, but also by the thermal regime of the roll, i.e. by the character of distribution of the established thermal conditions. As a result, the disturbing effect of the thermocouple was accounted for by using the two-model iterative algorithm proposed in [15].

Numerical calculations were performed on an ES-1045 computer. The machine time expended on analyzing one experiment by the conjugate gradient method was about 50 min with a step for the angular coordinate $\Delta\varphi=\pi/314$. The thermophysical properties of the materials of the roll and the thermocouple electrodes were taken from [16].

The result of determination of the function $q(\varphi)$ can be explained as follows. On the section where the skin is freezing, there is a gradual reduction in heat flux in the roll due to a reduction in the temperature difference between the roll surface and the skin and to separation of the skin from the surface. The subsequent sharp reduction in heat flux in the roll is connected with a decrease in the thermal resistance of the contact due to an increase in the pressure of the casting on the mold in the deformation zone.

In conclusion, we noted that the method described above can also be used to study the processes of ingot-rolling and cutting.

NOTATION

T, temperature field in the roll; T_{coo}, temperature of cooling liquid; T^*, inlet temperature; α, heat-transfer coefficient in convective cooling; a, λ, thermal conductivity and diffusivity of the roll; ω, angular velocity; $\bar{v} = \omega R^2/a$, dimensionless speed of rotation; $q(\varphi)$, heat flux on the roll surface; ρ_0, dimensionless internal radius of the roll.

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